Effects of supplemental viscous damping on inelastic seismic response of asymmetric systems

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SUMMARY

This paper investigates the effects of supplemental viscous damping on the seismic response of one-storey, asymmetric-plan systems responding in the inelastic range of behaviour. It was found that addition of the supplemental damping reduces not only deformation demand but also ductility and hysteretic energy dissipation demands on lateral load resisting elements during earthquake loading. However, the level of reduction strongly depends on the plan-wise distribution of supplemental damping. Nearly optimal reduction in demands on the outermost flexible-side element, an element generally considered to be the most critical element, was realized when damping was distributed unevenly in the system plan such that the damping eccentricity was equal in magnitude but opposite in algebraic sign to the structural eccentricity of the system. These results are similar to those noted previously for linear elastic systems, indicating that supplemental damping is also effective for systems expected to respond in the inelastic range. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: asymmetric buildings; earthquake response; inelastic response; passive control; protective systems; seismic response; supplemental damping; torsion; viscous damping

INTRODUCTION

Plan asymmetry has often been cited as the main cause for collapse of many buildings in past earthquakes [1]. Buildings located on street corners are prime candidates for large plan (torsional) irregularity. These buildings are often composed of windows on street frontages, and stiff infill masonry or concrete walls supported by moment frames on the remaining faces, resulting in a large stiffness eccentricity. The flexible-side lateral load-resisting elements, which are located on the open sides, often experience large deformation and energy dissipation demands during seismic events [2]. If these elements are not designed to accommodate the

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large demands, they may fail during a seismic event leading to the building's collapse. Large deformations may also cause pounding between closely spaced adjacent buildings and result in increased second-order (P–Δ) effects.

Several approaches may be used to reduce excessive earthquake-induced deformation, ductility, and hysteretic energy dissipation demands on lateral load resisting elements of asymmetric-plan systems. Seismic codes attempt to do so by providing additional strength to certain lateral load-resisting elements [3–5]. While this approach reduces ductility demand, it fails to control excessive deformation and hysteretic energy dissipation demands [2]. Another approach is to redistribute the stiffness and/or mass properties to minimize the stiffness eccentricity and hence adverse effects of torsion. While such approach is possible at an early design stage for some new structures, it may not be feasible for many other new structures because of architectural and/or functional constraints. It may not be feasible for existing structures because of the significant 'down time' and/or inconvenience to the occupants.

The approach of using supplemental damping is an appealing alternative to the other approaches. The addition of supplemental dampers to a structural system has been known to reduce the deformation and ductility demands as well as enhance its energy dissipation capacity [6–11]. However, most of past experience has been with planar (symmetric-plan) systems. Some recent studies have investigated the effects of plan-wise distribution of supplemental damping on seismic response of three-dimensional systems [12–14]. Using yielding devices with elastic–plastic force–deformation characteristics, Arista and Gomez [12] examined the effects of asymmetric distribution of supplemental dampers on seismic behaviour of single-storey systems. However, their study was limited to systems with symmetric plan. Martin and Pekau [13] and Pekau and Guimond [14] investigated the seismic response of asymmetric structures with friction dampers and found that such devices are effective in improving seismic performance of asymmetric-plan structures. It was also found that additional improvement in performance is obtained by properly 'tuning' the slip load distribution in the system's plan [13].

Fluid viscous dampers are especially attractive for enhancing the seismic performance of structures because they not only reduce the deformation demand but also the force demands. For example, a recent study by Constantinou and Symans [7] showed that the inclusion of fluid viscous dampers in the structures tested on a shake table resulted in reductions in storey drifts from 30 to 70 per cent. These reductions are comparable to those achieved by other supplemental damping devices. However, the use of fluid viscous dampers also resulted in reductions in storey shear forces by 40–70 per cent while other systems were incapable of achieving any comparable reduction. The reason for this difference is the nearly pure viscous behavior of the fluid dampers; the velocity-related forces resulting from viscous damping are nearly out-of-phase with the deformation-related forces.

Owing to the attractiveness of fluid viscous dampers for enhancing seismic performance of structures, an investigation was initiated by the first author on earthquake behaviour of linear, one-storey, asymmetric-plan systems with supplemental fluid viscous damping. The results of this investigation have been reported in a series of publications [15–17]. First, three system parameters were defined to account for the supplemental viscous damping: (1) the supplemental damping ratio, ζ_{sd}; (2) the normalized supplemental damping eccentricity, $\bar{e}_{sd}$ and (3) the normalized supplemental damping radius of gyration, $\bar{\rho}_{sd}$. Next, the effects of plan-wise distribution of supplemental viscous damping on seismic response were examined. It was found that plan-wise distribution of damping plays an important role in seismic behaviour

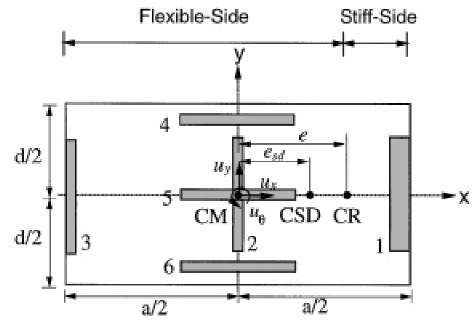


Figure 1. Idealized one-storey system.

of asymmetric-plan systems. In particular, the largest reduction in flexible-edge deformation was realized when the fluid viscous dampers were distributed in the system plan such that the damping eccentricity takes on the largest value with algebraic sign opposite to the structural eccentricity [15, 16]. Finally, various modal properties were examined and it was found that plan-wise distribution of supplemental viscous damping mainly influences the apparent modal damping ratios which in turn affect the deformation demands. Since the distribution which led to the largest damping eccentricity with algebraic sign opposite to the structural eccentricity led to the largest apparent damping ratio in the fundamental mode of vibration, it also led to the largest reduction in deformation of the flexible-edge [17].

While the aforementioned studies on elastic response of asymmetric-plan systems have led to an improved understanding of how supplemental viscous damping reduces earthquake-induced deformations, it is also important to study the response of structures responding in the inelastic range to understand how to design or enhance a structure so that damage is controlled at an acceptable level during intense ground shaking. For this purpose, the non-linear response of one-storey, asymmetric-plan systems with supplemental viscous damping was investigated. First, the elastic and inelastic system parameters necessary to control the response of one-storey, asymmetric-plan systems with supplemental viscous damping during earthquake loading are presented and the inelastic response quantities considered are defined. Next, the effects of supplemental damping are evaluated by comparing the inelastic response quantities of one-storey, asymmetric-plan systems with supplemental damping with those of the corresponding symmetric-plan system without supplemental damping. Finally, variations of demands with different combinations of structural eccentricity, damping eccentricity, and damping radius of gyration are studied to identify a near-optimal plan-wise distribution of supplemental damping that would minimize the demands on the flexible-side element.

SYSTEM AND GROUND MOTION

One-storey system

The model used for this study represents a one-storey building idealized as a rigid deck supported by six structural elements: three structural elements in each of the two orthogonal directions (Figure 1). The structural elements were frames or walls having strength and stiffness only in their planes. Fluid viscous dampers are incorporated into the bracing system. The mass properties of the system were assumed to be symmetric about the X- and Y-axis, thus the centre of mass (CM) coincided with its geometric center.

The stiffness and damper properties were considered to be symmetric only about the X-axis. The lack of symmetry in damping, about the Y-axis, was characterized by the supplemental damping eccentricity, e_{sd}, defined as the distance between the CM and the center of supplemental damping (CSD). The lack of symmetry in stiffness, about the Y-axis, was characterized by the stiffness eccentricity, e, defined as the distance between the CM and the center of rigidity (CR). The stiff edge of the system is defined as the edge that is on the same side of the CM as the CR; the other edge is the flexible edge (Figure 1). In the selected system, elements 1 and 3 are located on the stiff and flexible edges, respectively, and are denoted as the stiff-side and flexible-side elements, respectively.

The corresponding symmetric-plan system was defined as a system with no supplemental damping and coincidental CM and CR but with relative locations and stiffness of all resisting elements identical to those in the asymmetric-plan system.

Ground motion

The ground motion considered is the North–South (360°) component recorded at the Sylmar County Hospital parking lot during the 1994 Northridge earthquake. The peak values of the ground acceleration, velocity, and displacement recorded at the site were 826.6 cm/s^2, 128.9 cm/s, and 32.55 cm, respectively. This ground motion was applied to the system to act in the Y-direction.

SYSTEM PARAMETERS

Elastic system

The linear elastic response of one-storey, asymmetric-plan systems without supplemental damping depends on (1) transverse vibration period, $T_y = 2\pi/\omega_y$ (ω_y = vibration frequency), of the corresponding symmetric-plan system in the Y-direction; (2) normalized stiffness eccentricity, $\bar{e} = e/a$ (a = plan dimension perpendicular to the direction of ground motion); (3) ratio of the torsional and transverse frequencies, $_\theta$; (4) aspect ratio of the deck, $\alpha = a/d$ and (5) mass and stiffness proportional damping constants, a_0 and a_1, which in turn depend on the natural damping ratios in the two vibration modes of the system. The additional parameters needed to include supplemental damping are [15]: (1) supplemental damping ratio, ζ_{sd}; (2) normalized supplemental damping eccentricity, $\bar{e}_{sd} = e_{sd}/a$ and (3) normalized supplemental damping radius of gyration, $\bar{\rho}_{sd} = \rho_{sd}/a$. Detailed description of various parameters of a linear system is available elsewhere [15]. Parameters that characterize the inelastic system are discussed in the next section.

INELASTIC SYSTEM

Yield strength

The total yield strengths in the X- and Y-directions were calculated as

$$F_x = \frac{mA_x}{R_x} \quad \text{and} \quad F_y = \frac{mA_y}{R_y} \tag{1}$$

in which m is the system mass; R_x and R_y are the reduction factors in the X- and Y-directions, respectively, and A_x and A_y are the pseudo-accelerations for vibration periods T_x and T_y, respectively, selected from the mean $+\,1\sigma$ Newmark–Hall design spectrum. The Newmark–Hall design spectrum was constructed for 5 per cent damping and peak values of the ground acceleration, velocity and displacement equal to 826.6 cm/s^2, 128.9 cm/s, and 32.55 cm, respectively, using the procedure described in Chopra [18].

Element yield strength

For simplicity, the yield strength of various elements was assumed to be proportional to their stiffness. Therefore, the strength of the ith elements oriented in the Y-direction is computed by

$$f_y^i = \frac{F_y}{K_y} k_y^i \tag{2}$$

where F_y is the total yield strength of the system given by Equation (1), k_y^i is the stiffness of the ith element, and K_y is the total stiffness, all in the Y-direction. Similarly, the yield strength of the jth element oriented in the X-direction is calculated from

$$f_x^j = \frac{F_x}{K_x} k_x^j \tag{3}$$

where F_x is the yield strength of the system given by Equation (1), k_x^j is the stiffness of the jth element, and K_x is the total stiffness, all in the X-direction. The force–deformation behaviour of each resisting element was selected as elastic–plastic with 3 per cent post-yield strain hardening. The relationships defined by Equations (2) and (3) imply that the yield strength of an element in an asymmetric-plan system is identical to its yield strength in the corresponding symmetric-plan system.

The yield deformation of the ith element oriented in the Y-direction can be calculated as

$$u_y = \frac{f_y^i}{k_y^i} = \frac{F_y}{K_y} \tag{4}$$

and that of the jth element oriented in the X-direction as

$$u_x = \frac{f_x^j}{k_x^j} = \frac{F_x}{K_x} \tag{5}$$

Equations (4) and (5) indicate that for the selected strength distribution, yield deformation of all elements in a given direction are the same. Furthermore, the yield deformation of an element in the asymmetric-plan system is the same as the yield deformation of this element in the corresponding symmetric-plan system.

It is useful to note here that strength distribution selected in this investigation is consistent with the constant-D-type distribution described by Tso and Smith [19] and advocated by Paulay [20]. While other strength distributions are possible [3–5, 19], this distribution was selected for no other reason but simplicity.

SELECTED SYSTEM PARAMETERS

Responses are presented for the following values of system parameters. Values of T_y were selected in the range of 0.05–3 s, since damping is most effective in this period range [15, 17]. The selected value of $\ _{\theta} = 1$ represents systems with strong coupling between lateral and torsional motions. The value of $\ _{x} = 1$ corresponds to identical uncoupled vibration periods in the two orthogonal directions. The relative torsional stiffness parameter, γ_x, is given a value of 0.5 corresponding to an equal contribution to the system's torsional stiffness from the lateral resisting elements oriented along the two orthogonal directions. The normalized stiffness eccentricity, $\bar{e}$, was selected as 0.2, and the aspect ratio of the system, α, was fixed at two. The damping ratio, ζ, was fixed at 5 per cent in all modes of the corresponding linear elastic symmetric-plan system.

The supplemental damping ratio, ζ_{sd}, was fixed at 10 per cent. Three values were selected for the supplemental damping eccentricity, $\bar{e}_{\mathrm{sd}} = -0.2$, 0 and 0.2; $\bar{e}_{\mathrm{sd}} = -0.2$ corresponds to the CSD located at an equal distance from the CM as the CR but on the opposite side; $\bar{e}_{\mathrm{sd}} = 0$ corresponds to an even distribution of supplemental damping about the CM; and $\bar{e}_{\mathrm{sd}} = 0.2$ corresponds to coincidental locations of CR and CSD. The normalized supplemental damping radius of gyration, $\bar{\rho}_{\mathrm{sd}}$, was selected as 0.2 representing a medium spread of supplemental damping about the CSD. For the selected values of R_x and $R_y = 4$, the system was expected to be excited well into the inelastic range during the earthquake considered in this study. For selected cases, variations of $\bar{e}_{\mathrm{sd}}$ in the range of −0.5 to 0.5, $\bar{e}$ in the range of 0.0 to 0.5, $\bar{\rho}_{\mathrm{sd}}$ in the range of 0.0–0.5, and R_y in the range of 1–8, were also considered.

INELASTIC RESPONSE QUANTITIES CONSIDERED

Let us denote peak deformations of the stiff and flexible edge of an asymmetric-plan system as u_{s} and u_{f}, respectively, and peak deformation of the corresponding symmetric-plan system as u_0. Note that u_0 is also the deformation at the two edges of the corresponding symmetric-plan system because such a system undergoes no torsional motion. The ratio of the peak edge deformations in the asymmetric-plan and the corresponding symmetric-plan system is then defined as

$$\bar{u}_{\mathrm{s}} = \frac{u_{\mathrm{s}}}{u_0} \quad \text{and} \quad \bar{u}_{\mathrm{f}} = \frac{u_{\mathrm{f}}}{u_0} \tag{6}$$

The normalized edge deformations, $\bar{u}_{\mathrm{s}}$ and $\bar{u}_{\mathrm{f}}$, are indicative of the effects of plan asymmetry. Since elements 1 and 3 are located at the stiff and flexible edges, respectively, deformations of these elements are equal to u_{s} and u_{f}, respectively, and the normalized element deformations are given by

$$\bar{u}_1 = \bar{u}_{\mathrm{s}} \quad \text{and} \quad \bar{u}_3 = \bar{u}_{\mathrm{f}} \tag{7}$$

Ductility demand on a lateral load-resisting element is defined as its peak deformation divided by its yield deformation. Therefore, ductility demands on elements 1 and 3 in the asymmetric-plan system is given as

$$\mu_1 = \frac{u_1}{u_y} \quad \text{and} \quad \mu_3 = \frac{u_3}{u_y} \tag{8}$$

and on the same elements of the corresponding symmetric-plan system as

$$\mu_0 = \frac{u_0}{u_y} \tag{9}$$

The normalized ductility demands on the two elements are then defined as

$$\bar{\mu}_1 = \frac{\mu_1}{\mu_0} = \left(\frac{u_1}{u_y}\right) \Big/ \left(\frac{u_0}{u_y}\right) = \frac{u_1}{u_0} = \bar{u}_1 \tag{10a}$$

$$\bar{\mu}_3 = \frac{\mu_3}{\mu_0} = \left(\frac{u_3}{u_y}\right) \Big/ \left(\frac{u_0}{u_y}\right) = \frac{u_3}{u_0} = \bar{u}_3 \tag{10b}$$

Combining Equations (6), (7) and (10) gives

$$\bar{u}_{\mathrm{s}} = \bar{u}_1 = \bar{\mu}_1 \quad \text{and} \quad \bar{u}_{\mathrm{f}} = \bar{u}_3 = \bar{\mu}_3 \tag{11}$$

Since deformations of an edge and of an element located at this edge are the same, both these deformations will be referred to by a single term as ‘element deformation’ in the rest of this paper.

Let E_{D} and E_{D0} denote the total energy dissipated through damping in the asymmetric-plan and its corresponding symmetric-plan systems, respectively. The normalized value of the dissipated energy is defined as

$$\bar{E}_{\mathrm{D}} = \frac{E_{\mathrm{D}}}{E_{\mathrm{D0}}} \tag{12}$$

Similarly, let E_{H} and E_{H0} denote the total hysteretic energy dissipated by all lateral load-resisting elements in the asymmetric-plan and its corresponding symmetric-plan systems, respectively. The normalized values of the hysteretic energy is then defined as

$$\bar{E}_{\mathrm{H}} = \frac{E_{\mathrm{H}}}{E_{\mathrm{H0}}} \tag{13}$$

The normalized hysteretic energy dissipated by the stiff and flexible elements are defined as

$$\bar{E}_{\mathrm{H1}} = \frac{E_{\mathrm{H1}}}{E_{\mathrm{H0}}} \quad \text{and} \quad \bar{E}_{\mathrm{H3}} = \frac{E_{\mathrm{H3}}}{E_{\mathrm{H0}}} \tag{14}$$

respectively, where E_{H1} and E_{H3} are the hysteretic energy dissipated by the stiff and flexible elements, respectively.

In this investigation, variations of the normalized response quantities $\bar{u}_{\mathrm{s}} = \bar{u}_1 = \bar{\mu}_1$, $\bar{u}_{\mathrm{f}} = \bar{u}_3 = \bar{\mu}_3$, $\bar{E}_{\mathrm{D}}, \bar{E}_{\mathrm{H}}, \bar{E}_{\mathrm{H1}}$ and $\bar{E}_{\mathrm{H3}}$ with various system parameters are examined.

FORCE–DEFORMATION HISTORIES

In order to understand how the presence of supplemental damping influences the hysteretic behaviour of lateral load-resisting elements during an earthquake, force–deformation histories for the flexible- and stiff-side elements for an asymmetric-plan system ($\bar{e} = 0.2$, ${}_{\theta} = \Omega_x = 1$, $\gamma_x = 0.5, \alpha = 2$, $\zeta = 0.05$, and $R_y = R_x = 4$) with supplemental damping ($\zeta_{\mathrm{sd}} = 0.1$, $\bar{e}_{\mathrm{sd}} = -0.2$, and $\bar{\rho}_{\mathrm{sd}} = 0.2$) are compared to those for the same elements in a system without supplemental

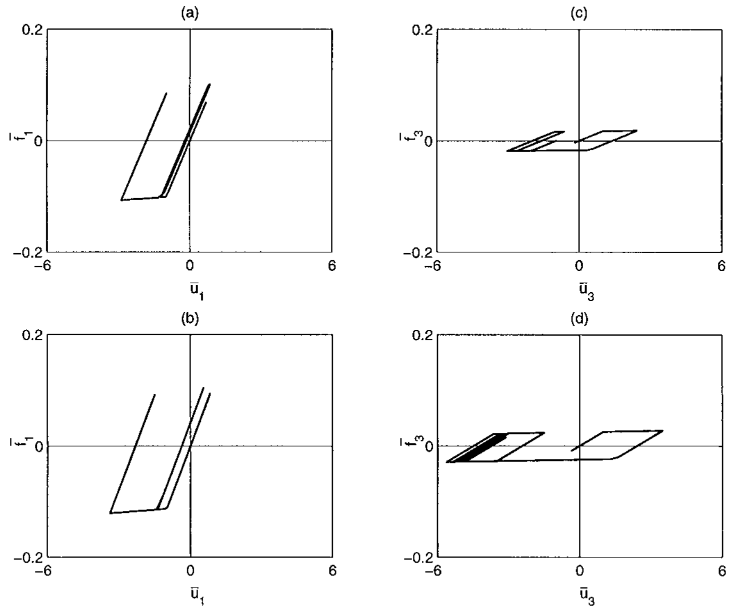


Figure 2. Force–deformation histories of resisting elements due to the 1994 Northridge earthquake, $T_y = 0.5$ s: (a) stiff-side element, $\zeta_{sd} = 0.1$; (b) stiff-side element, $\zeta_{sd} = 0$; (c) flexible-side element, $\zeta_{sd} = 0.1$; (d) flexible-side element, $\zeta_{sd} = 0$.

damping ($\zeta_{sd} = 0$). The results are presented for two period values, $T_y = 0.5$ and 1 s, and $R_x = R_y = 4$ in Figures 2 and 3. In these figures, the element force is normalized with the total yield force of the system in the Y-direction, $\bar{f}_i = f_i/F_y$, and the element deformation is normalized with the system yield deformation in the Y-direction, $\bar{u}_i = u_i/u_y$.

The presented results show that the flexible-side element in a short period ($T_y = 0.5$ s) asymmetric-plan system with supplemental damping undergoes inelastic cycles with significantly smaller deformation magnitudes than in a system without supplemental damping (Figures 2(c) and 2(d)). Furthermore, the hysteretic energy demand, represented by the area within the force–deformation loop, is much smaller in the flexible-side element of the system with supplemental damping compared to that in the system without supplemental damping. This indicates that supplemental damping significantly reduces the deformation and energy dissipation demand on the flexible-side element. The effects of supplemental damping on the stiff-side element for a short period system (Figures 2(a) and 2(b)) are minimal. The trends for a longer period system ($T_y = 1$ s) (Figure 3) are similar to those observed for the short period system ($T_y = 0.5$ s) with the exception that the stiff-side element does not yield in both systems, with and without supplemental damping.

MAXIMUM DEFORMATION AND DUCTILITY

The effects of supplemental damping are evaluated next by comparing the normalized deformations and ductility demands, $\bar{u}_s = \bar{u}_1 = \bar{\mu}_1$ and $\bar{u}_f = \bar{u}_3 = \bar{\mu}_3$, of a system with supplemental

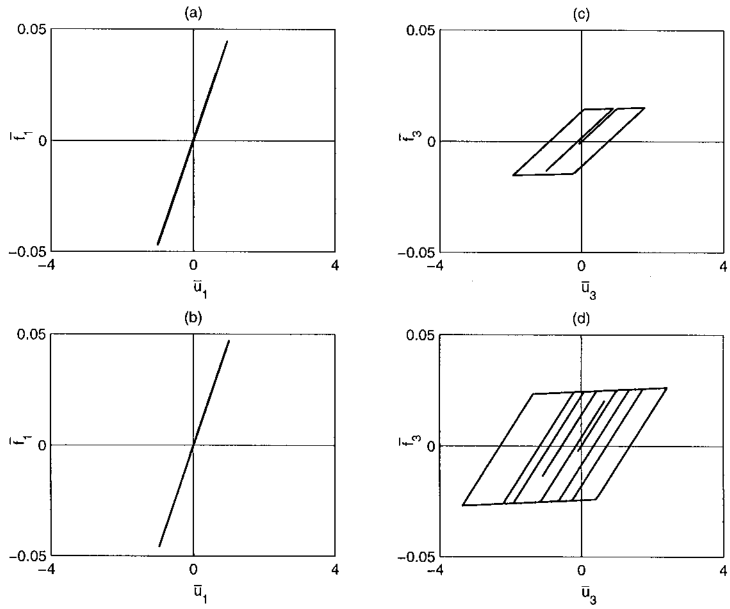


Figure 3. Force–deformation histories of resisting elements due to the 1994 Northridge earthquake, $T_y = 1.0\,\mathrm{s}$: (a) stiff-side element, $\zeta_{sd} = 0.1$; (b) stiff-side element, $\zeta_{sd} = 0$; (c) flexible-side element, $\zeta_{sd} = 0.1$; (d) flexible-side element, $\zeta_{sd} = 0$.

damping ($\zeta_{sd} = 0.1$) to those of the same system without supplemental damping ($\zeta_{sd} = 0$). Figure 4 presents the variation of the normalized element deformation and ductility with period T_y in asymmetric-plan systems ($\bar{e} = 0.2$, $_\theta = \Omega_x = 1$, $\gamma_x = 0.5$, $\alpha = 2$, $\zeta = 0.05$, and $R_y = R_x = 4$) with supplemental damping ($\zeta_{sd} = 0.1$ and $\bar{\rho}_{sd} = 0.2$) for three values of $\bar{e}_{sd} = -0.2$, 0, and 0.2 along with the asymmetric-plan system without supplemental ($\zeta_{sd} = 0$). These results show that supplemental damping has the effect of reducing deformations and ductility demands of both elements. However, the effect strongly depends on the plan-wise distribution of supplemental damping. For the stiff-side element (Figure 4(a)), $\bar{e}_{sd} = 0.2$ led to the largest reduction and $\bar{e}_{sd} = -0.2$ led to the smallest reduction in deformation and ductility. For the flexible-side element (Figure 4(b)), $\bar{e}_{sd} = -0.2$ led to the largest reduction, whereas $\bar{e}_{sd} = 0.2$ led to the smallest reduction. A uniform distribution of supplemental damping, $\bar{e}_{sd} = 0$, led to an intermediate reduction for both edges. These effects are much less pronounced for the stiff-side element compared to those for the flexible-side element, as apparent from closeness of the three curves for $\bar{e}_{sd} = -0.2$, 0 and 0.2 (Figure 4(a)).

The presented results show that supplemental damping may reduce the element deformations by a factor of nearly 2. For example, for a system with a period of 1 s, deformations of the flexible-side element is reduced by a factor of about 1.8; $\bar{u}_3 = 2.2$ and 1.25 for the system with supplemental damping ($e_{sd} = -0.2$) and without supplemental damping ($\bar{\zeta}_{sd} = 0$), respectively. The above-noted effects are similar to those noted earlier for linearly elastic systems [15], indicating that supplemental damping is effective even for systems responding in the inelastic range of behaviour.

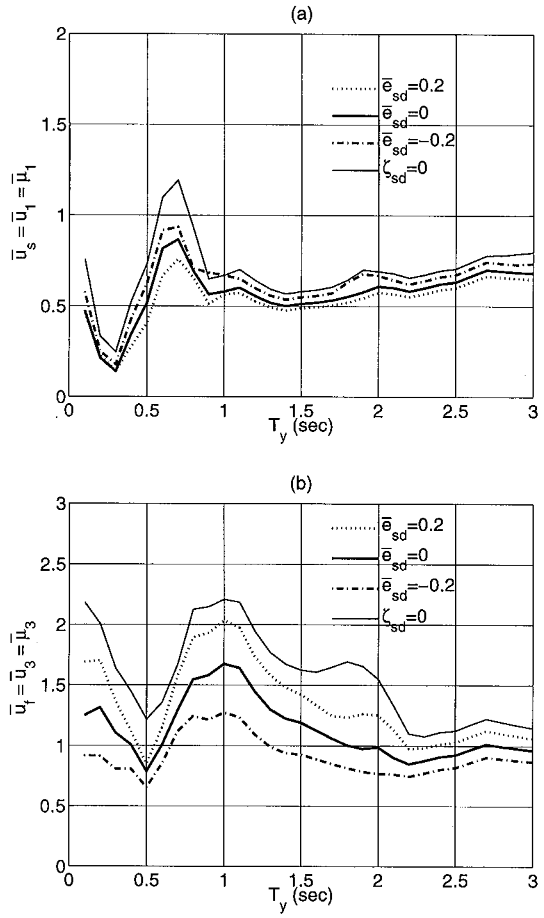


Figure 4. Normalized responses in asymmetric-plan systems with and without supplemental damping: (a) stiff-side element and (b) flexible-side element.

With proper distribution of supplemental damping, the demand in the flexible-side element can be reduced to nearly that of the corresponding symmetric-plan system. This becomes apparent for values of $\bar{u}_{\mathrm{f}} = \bar{u}_3 = \bar{\mu}_3$ for $\bar{e}_{\mathrm{sd}} = -0.2$ (Figure 4(b)), which are very close to one over the entire period range. Note that values of $\bar{u}_{\mathrm{f}} = \bar{u}_3 = \bar{\mu}_3$ larger than one indicate that the demand in the flexible-side element of the asymmetric-plan system is higher than that of the same element in the corresponding symmetric-plan system. Conversely, $\bar{u}_{\mathrm{f}} = \bar{u}_3 = \bar{\mu}_3$ smaller than one indicate that the demand in the flexible-side element of the asymmetric-plan system is lower than that of the same element in the corresponding symmetric-plan system.

The above results indicate that supplemental damping, with proper plan-wise distribution, is very effective for controlling the excess deformation and ductility demands on the flexible-side element. In general, the code-based approach tends to control the excess ductility demand on this element in the asymmetric-plan system by providing additional strength to vulnerable elements, e.g. through design eccentricity concept [3, 4]. The approach of using supplemental damping may be especially appealing for rehabilitation of systems for which adding strength may not be either economically or physically feasible.

The results presented so far are for a fixed value of the strength reduction factor, $R_x = R_y = 4$. It would be useful to investigate if the conclusions based on a single value of the reduction factor are applicable over a broad range of inelastic action, i.e., several values of the reduction factor. For this purpose, normalized deformation and ductility demands were computed for the flexible- and stiff-side elements of the asymmetric-plan system ($\bar{e} = 0.2$, ${}_{\theta} = \Omega_x = 1$, $\gamma_x = 0.5$, $\alpha = 2$, and $\zeta = 0.05$) with supplemental damping ($\zeta_{sd} = 0.1$, $\bar{e}_{sd} = -0.2$, and $\bar{\rho}_{sd} = 0.2$) for values of R_y ranging from 1 to 10 while keeping R_x fixed at 4; $R_y = 1$ corresponds to little or no inelastic action and $R_y = 10$ corresponds to significant inelastic action. Since the effects of inelastic action are known to be strongly dependent on the period region [18], three values of $T_y = 0.5, 1.0$, and $3.0\,\mathrm{s}$ were considered. These periods represent short-, medium-, and long-period systems, respectively.

The presented results (Figure 5) indicate that variations of normalized deformations and ductilities for medium-period ($T_y = 1\,\mathrm{s}$) and long-period systems ($T_y = 3\,\mathrm{s}$) over the considered R_y range are minimal (no more than 25 per cent). This indicates that trends observed earlier based on $R_y = 4$ are applicable to systems with other values of R_y. However, for the short-period system ($T_y = 0.5\,\mathrm{s}$), the normalized deformations and ductilities in systems with $R_y > 4$ can be significantly higher compared to those for $R_y = 4$. For example, $\bar{u}_3$ for $R_y = 7$ is nearly twice that for $R_y = 4$; $\bar{u}_3 = 1.14$ for $R_y = 7$ and 0.65 for $R_y = 4$. Therefore, much higher deformations and ductilities may be expected in short-period systems with $R_y > 4$ than those predicted based on $R_y = 4$. For values of $R_y < 4$ the trends are similar to those for $R_y = 4$, indicating that previously observed trends are applicable for $R_y < 4$.

DAMPING AND HYSTERETIC ENERGIES

The variations of normalized damping and hysteretic energies, $\bar{E}_D$ and $\bar{E}_H$, with period T_y are presented in Figure 6 for asymmetric-plan systems ($\bar{e} = 0.2$, ${}_{\theta} = \Omega_x = 1$, $\gamma_x = 0.5$, $\alpha = 2$, $\zeta = 0.05$, and $R_y = R_x = 4$) with supplemental damping ($\zeta_{sd} = 0.1$ and $\bar{\rho}_{sd} = 0.2$) for three values of $\bar{e}_{sd} = -0.2, 0$, and 0.2 and without supplemental damping ($\zeta_{sd} = 0$). As expected, $\bar{E}_D$ is larger (Figure 6(a)) and $\bar{E}_H$ is smaller (Figure 6(b)) in asymmetric-plan systems with supplemental damping than those without supplemental damping ($\zeta_{sd} = 0$). Furthermore, short-period systems (e.g. $T_y < 0.5\,\mathrm{s}$) dissipate a smaller fraction of the total energy through damping and a larger fraction through hysteretic action compared to longer period systems.

The presented results show that the largest values of $\bar{E}_D$ (Figure 6(a)) tend to occur for $\bar{e}_{sd} = -0.2$, whereas the system without supplemental damping led to the smallest values of $\bar{E}_D$; $\bar{E}_D$ is not very sensitive to the plan-wise distribution of supplemental damping as apparent from closeness of the curves for three values of $\bar{e}_{sd}$. For systems with T_y up to about $2\,\mathrm{s}$, $\bar{e}_{sd} = -0.2$ led to the smallest $\bar{E}_H$. Since the hysteretic energy is dissipated through inelastic action that is associated with damage in the system, the presented results indicate that plan-wise distribution of supplemental damping with $\bar{e}_{sd} = -0.2$ would lead to smaller damage in such systems. For $T_y > 2\,\mathrm{s}$, the trend reverses and $\bar{e}_{sd} = 0.2$ led to the smallest values of $\bar{E}_H$.

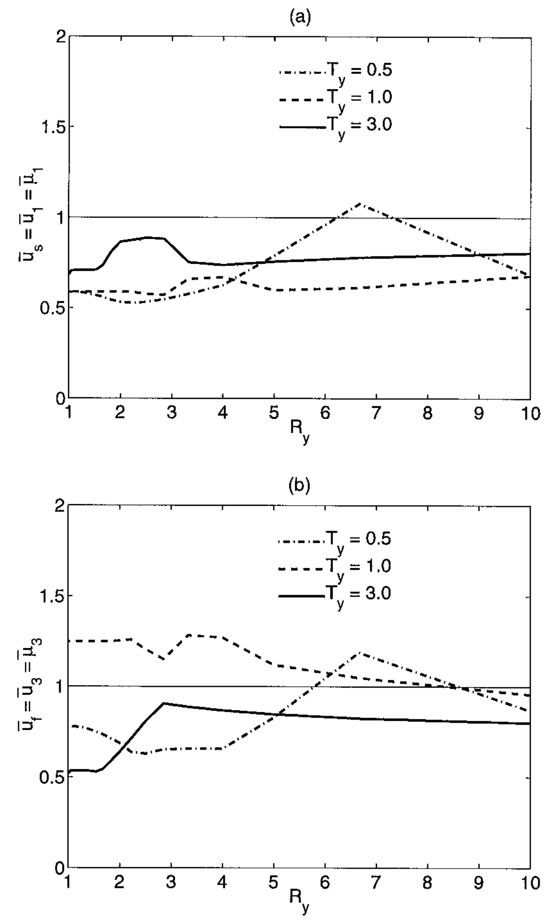


Figure 5. Variation of normalized response with strength reduction factor for asymmetric-plan systems: (a) stiff-side element and (b) flexible-side element.

Note that the period value at which the reversal in trend occurs is likely to depend on the input ground motion.

The variation of normalized hysteretic energy for the stiff-side element, $\bar{E}_{\mathrm{H1}}$, and flexible-side element, $\bar{E}_{\mathrm{H3}}$, with period T_y is presented in Figure 7; system parameters selected are the same as those mentioned previously for Figure 6. These results show that the largest reduction in hysteretic energy for the flexible-edge element (Figure 7(b)) is obtained for $\bar{e}_{\mathrm{sd}} = -0.2$. For example, in a system with $T_y = 0.9$, the hysteretic energy on the flexible-side element was reduced by a factor of nearly 4 with supplemental damping distributed such that $\bar{e}_{\mathrm{sd}} = -0.2$; $\bar{E}_{\mathrm{H3}} = 0.35$ for a system with $\zeta_{\mathrm{sd}} = 0.1$, $\bar{\rho}_{\mathrm{sd}} = 0.2$, and $\bar{e}_{\mathrm{sd}} = -0.2$ and $\bar{E}_{\mathrm{H3}} = 1.45$ for a system with $\zeta_{\mathrm{sd}} = 0$. Consistent with the previous observations on deformations, the largest reduction in hysteretic energy for the stiff-side element occurs for $\bar{e}_{\mathrm{sd}} = 0.2$ (Figure 7(a)). Furthermore, the sensitivity of $\bar{E}_{\mathrm{H3}}$ to plan-wise distribution of supplemental

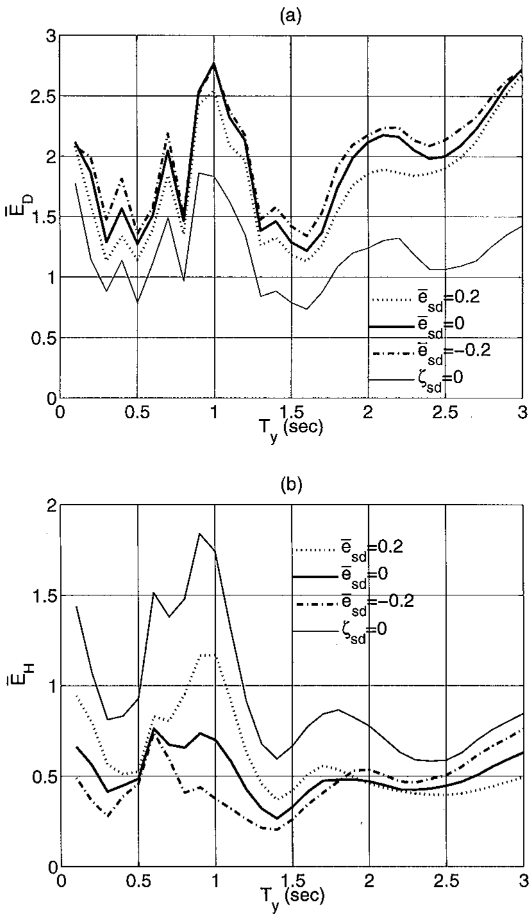


Figure 6. (a) Total normalized damping energy and (b) total normalized hysteretic energy for systems with and without supplemental damping.

damping decreases, whereas that of $\bar{E}_{H1}$ increases as the system period increases in the range of $T_y > 1.5$ s.

Since reduction in hysteretic energy corresponds to reduction in damage due to inelastic action, as noted previously, the presented results (Figure 7) indicate that supplemental damping can be used very effectively in reducing damage in the flexible-side element, the most vulnerable element, of asymmetric-plan systems.

OPTIMAL PLAN-WISE DISTRIBUTION OF SUPPLEMENTAL DAMPING

Results presented so far indicate that supplemental viscous damping can be used to reduce deformations, ductility, and damage in asymmetric-plan systems responding in the inelastic range. The level of reduction, however, depends on the plan-wise distribution of supplemental

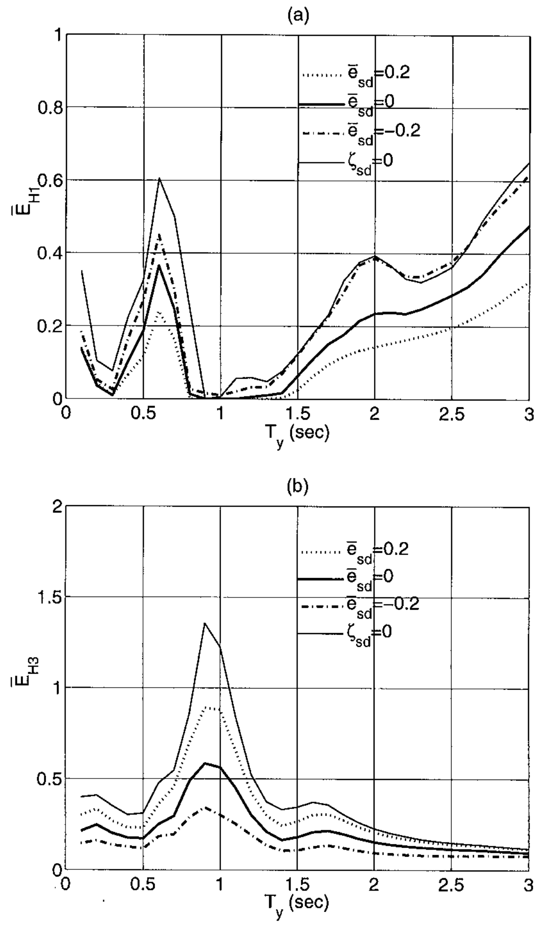
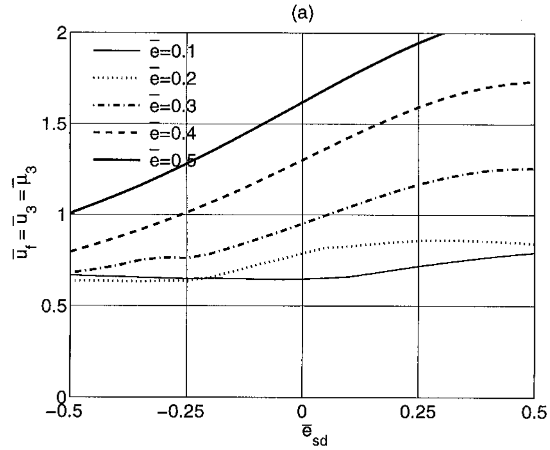


Figure 7. Normalized hysteretic energy for (a) stiff-side element and (b) flexible-side element in systems with and without supplemental damping.

damping. It would be useful to establish the plan-wise distribution of supplemental damping that leads to nearly the largest (or near-optimal) reduction in the deformation and ductility demands. For this purpose variation of deformation and ductility with $\bar{e}_{sd}$ for several values of structural eccentricity, $\bar{e}$, ranging from 0.1 to 0.5, are plotted in Figures 8 and 9 ($\zeta_{sd}=0.1$, $\bar{\rho}_{sd}=0.2$, $_{\theta}=\Omega_x=1$, $\gamma_x=0.5$, $\alpha=2$, $\zeta=0.05$, and $R_y=R_x=4$); and for three values of $\bar{\rho}_{sd}=0, 0.2$, and 0.5 are presented in Figures 10 and 11 ($\zeta_{sd}=0.1$, $\bar{e}=0.2$, $_{\theta}=\Omega_x=1$, $\gamma_x=0.5$, $\alpha=2$, $\zeta=0.05$, and $R_y=R_x=4$). Two values of $T_y=0.5$ and 1 s are considered, representing short-period and medium-period systems, respectively. The values of $\bar{e}_{sd}$ are varied from -0.5 to 0.5. The extreme values of $\bar{e}_{sd}=-0.5$ and 0.5 correspond to all dampers located on the flexible and stiff edge, respectively.

Figure 8 shows that regardless of the structural eccentricity, $\bar{e}$, the smallest deformation and ductility of the flexible-side element occurs for $\bar{e}_{sd}=-0.5$, i.e. when all dampers are



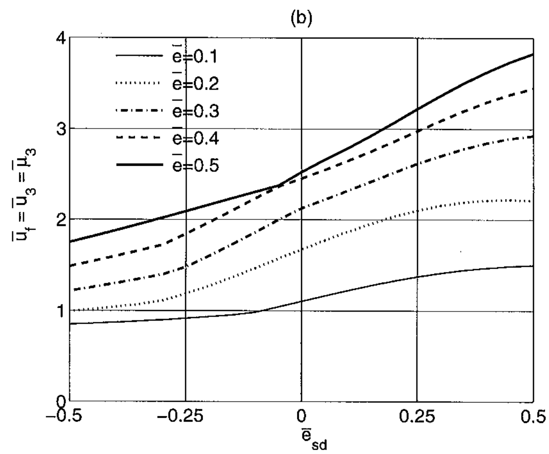


Figure 8. Variation of normalized response for flexible-side element with supplemental damping eccentricity for five values of $\bar{e}$: (a) $T_y = 0.5$ s and (b) $T_y = 1.0$ s.

concentrated at the flexible edge. Deformation and ductility are the largest for $\bar{e}_{sd} = 0.5$ and decrease as $\bar{e}_{sd}$ varies from 0.5 to -0.5, i.e. as the CSD moves from the stiff edge to the flexible edge. The curves flatten as $\bar{e}_{sd}$ approaches -0.5 and the flattening starts approximately at $\bar{e}_{sd} = -\bar{e}$. This indicates that the nearly smallest normalized responses occur for $\bar{e}_{sd} = -\bar{e}$; additional reductions in the range of $\bar{e}_{sd} = -\bar{e}$ to -0.5 are minimal.

For a system with fixed $\bar{e}_{sd}$, the normalized responses increase with $\bar{e}$, as may be expected due to increasing plan asymmetry. These trends are similar for both values of $T_y = 0.5$ (Figure 8(a)) and 1 s (Figure 8(b)).

The trends for the stiff-side element (Figure 9) are reversed compared to the flexible-side element (Figure 8). The smallest deformation occurs for $\bar{e}_{sd} = 0.5$, i.e. when all dampers are concentrated at the stiff edge; and for a system with fixed $\bar{e}_{sd}$, normalized responses decrease with increasing $\bar{e}$. The effects are, however, less pronounced for the stiff-side element compared to the flexible-side element.

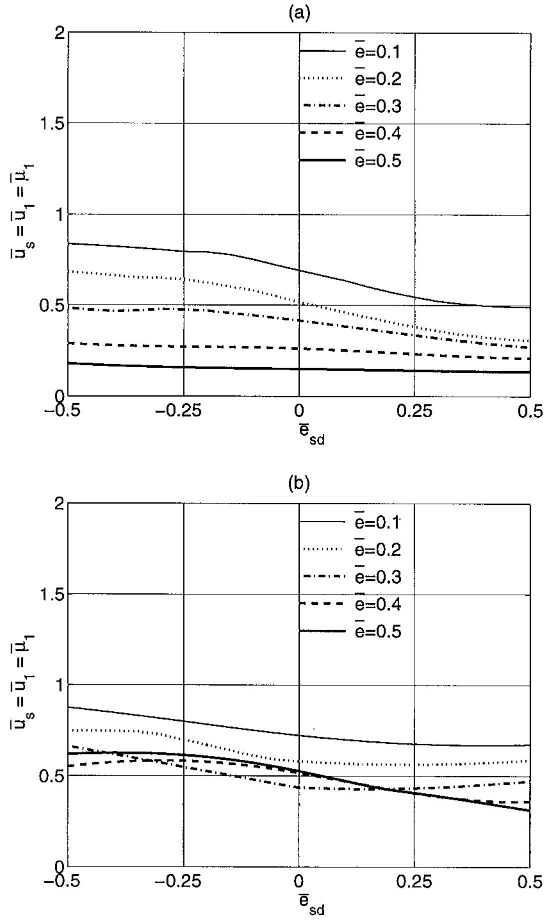


Figure 9. Variation of normalized response for stiff-side element with supplemental damping eccentricity for five values of $\bar{e}$: (a) $T_y = 0.5$ s and (b) $T_y = 1.0$ s.

Figure 10 shows that over a wide range of $\bar{e}_{sd}$, largest values of $\bar{\rho}_{sd}$ leads to the smallest values of the normalized responses. In the range of $\bar{e}_{sd}$ from -0.25 to -0.5, however, the system response is insensitive to $\bar{\rho}_{sd}$, as apparent from the closeness of all curves; for $T_y = 0.5$ s there is cross over of the curves but all curves are still very close. The normalized response of the stiff-side element (Figure 11) shows very little dependence on $\bar{\rho}_{sd}$ over the entire range of $\bar{e}_{sd}$ as all curves are very close. The trends for the two T_y values are very similar except for minor differences such as cross over of curves.

Since one of the major concerns for asymmetric-plan buildings is to reduce deformation and hysteretic energy dissipation demands on the flexible-side element, the optimal plan-wise distribution of the supplemental damping is the one that leads to the smallest demands on this element. The results presented in this section indicate that nearly the smallest demands on this element are obtained at about $\bar{e}_{sd} = -\bar{e}$; additional reductions, although possible between

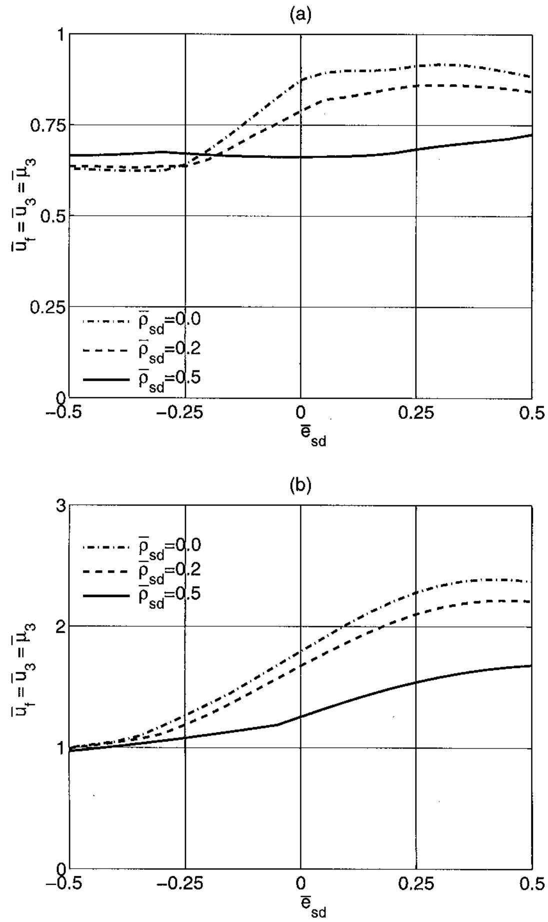


Figure 10. Variation of normalized response for flexible-side element with supplemental damping eccentricity for three values of $\bar{\rho}_{sd}$: (a) $T_y = 0.5$ s and (b) $T_y = 1.0$ s.

$\bar{e}_{sd} = -\bar{e}$ and -0.5, are minimal. In this range, deformations of the flexible edge are insensitive to $\bar{\rho}_{sd}$. Therefore, the optimal plan-wise distribution of supplemental damping corresponds to $\bar{e}_{sd} = -\bar{e}$, i.e. the CSD located at a distance equal to the structural eccentricity from the centre of mass towards the flexible edge.

CONCLUSION

This investigation on the seismic response of one-storey, asymmetric-plan systems responding in the inelastic range of behaviour, has led to the following conclusions:

- Supplemental viscous damping can be used to reduce deformation, ductility, and hysteretic energy dissipation demands in lateral load-resisting elements of asymmetric-plan systems

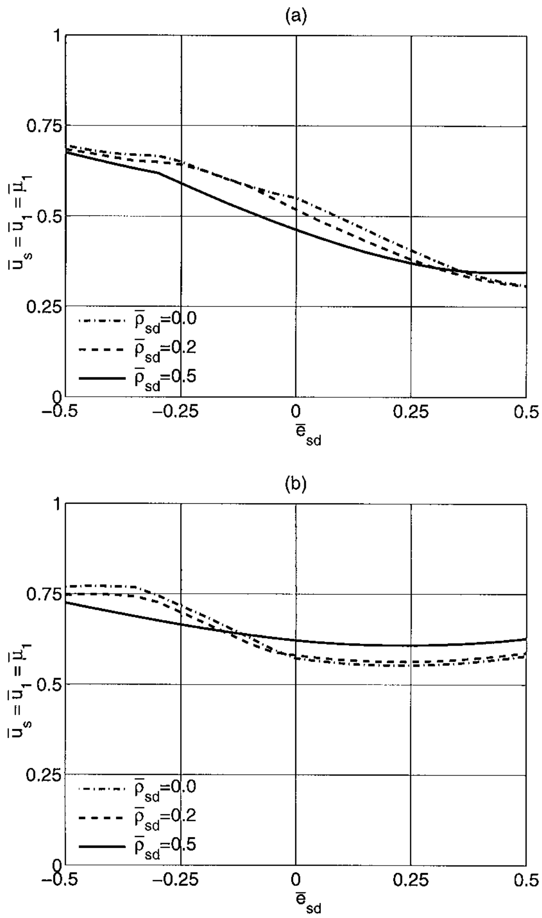


Figure 11. Variation of normalized response for stiff-side element with supplemental damping eccentricity for three values of $\bar{\rho}_{sd}$: (a) $T_y = 0.5$ s and (b) $T_y = 1.0$ s.

responding in the inelastic range. However, the level of reduction strongly depends on the plan-wise distribution of the supplemental damping.

- With proper distribution of supplemental damping, the deformation and ductility demands in the flexible-side element can be reduced to nearly that of the corresponding symmetric-plan system.
- The trends based on $R_y = 4$, the reduction factor for which most results are presented in this paper, are applicable to systems with other values of R_y for medium- and long-period systems. For the short-period system ($T_y = 0.5$ s), however, higher deformation and ductility demands may occur for larger values of R_y. The overall trends are, however, not affected by the degree of inelastic action, i.e. values of R_y.

- The near optimal plan-wise distribution of supplemental damping for reducing demands on the flexible-side element, generally considered to be the most critical element in asymmetric-plan systems, occurs when $\bar{e}_{sd} = -\bar{e}$, i.e. the CSD located at a distance equal to the structural eccentricity from the centre of mass towards the flexible edge. Such a distribution of supplemental viscous damping leads to the smallest deformation and ductility demands on this element.

Although the results are presented in this paper for a single ground motion, these observations were found to be valid for other ground motions. In particular, results were verified for the ground motion recorded at the El Centro site during the 1940 Imperial Valley earthquake, as well as for a suite of 20 ground motions developed for the SAC project.

The results presented in this study are for systems subjected to only one component of the ground motion (in the Y-direction). It would be useful to examine how the trends differ when the system is subjected to two orthogonal components of the ground motion simultaneously. Study along these lines is currently underway.

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